

On the Siciak extremal function for real compact convex sets

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1. Introduction

Let E be a bounded Borel set in $\mathbf{C}^N$. Define

$$(1.1) \quad V_E(z) := \sup\{u(z) : u \in L, \ u \leq 0 \text{ on } E\},$$

where

$$L := \{u \text{ plurisubharmonic in } \mathbf{C}^N : u(z) \leq \log^+ |z| + C \text{ for some } C\}$$

is the class of plurisubharmonic functions of logarithmic growth (here we have $|z| = (\sum_{j=1}^N |z_j|^2)^{1/2}$ and $\log^+ |z| = \max\{0, \log |z|\}$). Then the upper semicontinuous regularization $V_E^*(z) := \limsup_{\zeta \rightarrow z} V_E(\zeta)$ is called the (Siciak) extremal function of E . If K is a compact set in $\mathbf{C}^N$, then the extremal function in (1.1) can be gotten via the formula

$$(1.2) \quad V_K(z) := \max\left\{0, \sup\left\{\frac{1}{\deg p} \log |p(z)| : p \text{ holomorphic polynomial, } \|p\|_K \leq 1\right\}\right\}$$

(Theorem 5.1.7 in [K1]). Here, $\|p\|_K := \sup_{z \in K} |p(z)|$ denotes the uniform norm on K . We say that K is regular if and only if $V_K^* = V_K$. Note that if we let

$$\hat{K} := \{z \in \mathbf{C}^N : |p(z_1, \dots, z_N)| \leq \|p\|_K \text{ for all polynomials } p\}$$

denote the polynomial hull of K , then

1. $\hat{K} = \{z \in \mathbf{C}^N : V_K(z) = 0\}$;
2. $V_{\hat{K}} = V_K$.