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Very weak solutions of parabolic systems of p-Laplacian type

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Abstract. We show that the standard assumptions on weak solutions to certain parabolic systems can be weakened and still the usual regularity properties of solutions can be obtained. In order to do this, we derive estimates for the solutions below the natural exponent and then apply reverse Hölder inequalities.

1. Introduction

Our work is motivated by the classical Weyl's lemma: If a locally integrable function satisfies Laplace's equation in the sense of distributions, then it is real analytic. In other words, only a very modest requirement on the regularity of a solution is needed for a partial differential equation to make sense and then the equation gives extra regularity. We are interested in nonlinear parabolic systems of partial differential equations so that a counterpart of Weyl's lemma is too much to hope for, but the question of relaxing the standard Sobolev type assumptions on weak solutions and still obtaining regularity theory is the objective of our work.

We consider solutions to second order parabolic systems

$$\frac{\partial u_i}{\partial t} = \operatorname{div} A_i(x,t,\nabla u) + B_i(x,t,\nabla u), \quad i=1,\dots,N. \tag{1.1}$$

In particular, we are interested in systems of p-Laplacian type. The principal prototype is the p-parabolic system

$$\frac{\partial u_i}{\partial t} = \operatorname{div}(|\nabla u|^{p-2}\nabla u_i), \quad i=1,\dots,N,$$

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