# Weyl multipliers, Bochner—Riesz means and special Hermite expansions

S. Thangavelu

*Dedicated to Prof. K. G. Ramanathan on his 70th birthday*

## 1. Introduction

Let $\hat{f}$ denote the Fourier transform of $f$ on $\mathbf{R}^n$. Let $B_R^\delta$ be the Bochner—Riesz means for the Laplacian on $\mathbf{R}^n$ defined by

$$(B_R^\delta f)^\wedge(\xi) = \left(1-\frac{|\xi|^2}{R^2}\right)_+^\delta \hat{f}(\xi). \tag{1.1}$$

These operators are related to the summability of Fourier integrals and multiple Fourier series. It is well-known that these operators are not uniformly bounded on $L^p(\mathbf{R}^n)$ unless $p$ lies in the interval $\frac{2n}{n+1+2\delta}<p<\frac{2n}{n-1-2\delta}$. When $\delta>\frac{n-1}{2}$, these operators are uniformly bounded on all $L^p(\mathbf{R}^n)$, for $1\leqq p\leqq\infty$. When $0<\delta\leqq\frac{n-1}{2}$, the conjecture is that $B_R^\delta$ are uniformly bounded if $p$ lies in the above interval. When $n=2$ the conjecture is proved and when $n>2$ it is only proved for $\delta>\frac{n-1}{2(n+1)}$ (see [2]). Moreover, in a celebrated work [3] C. Fefferman has shown that the conjecture is false when $\delta=0$.

In this paper we like to treat a similar problem for the Weyl transform. The Weyl transform $W$ takes functions on $\mathbf{C}^n$ into operators bounded on $L^2(\mathbf{R}^n)$. $W$ enjoys many properties of the Fourier transform and is closely related to expansions in terms of Laguerre, Hermite and special Hermite functions. So it will be interesting to study multipliers for the Weyl transform analogous to Fourier multipliers. In [8] Mauceri has studied general multipliers for the Weyl transform. In [15] the author