

# Classification of Levi degenerate homogeneous CR-manifolds in dimension 5

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## Contents

1. Introduction . . . . .	1
Part I. Levi degenerate CR-manifolds . . . . .	8
2. Preliminaries . . . . .	8
3. Tube manifolds . . . . .	11
4. Tube manifolds over cones . . . . .	15
5. Some examples . . . . .	21
6. Levi degenerate CR-manifolds associated with an endomorphism . . . . .	25
7. Homogeneous 2-nondegenerate manifolds of CR-dimension 2 . . . . .	33
8. Homogeneous 2-nondegenerate CR-manifolds in dimension 5 . . . . .	40
Part II. The classification . . . . .	44
9. Lie-theoretic characterization of locally homogeneous CR-manifolds . . . . .	44
10. The Lie algebra $\mathfrak{g}$ has small semisimple part . . . . .	49
11. The cases $\mathfrak{s} \cong \mathfrak{so}(2, 3)$ and $\mathfrak{s} \cong \mathfrak{so}(1, 3)$ . . . . .	54
12. The case $\mathfrak{s} \cong \mathfrak{sl}(2, \mathbb{R})$ . . . . .	57
13. The case $\mathfrak{s} \cong \mathfrak{su}(2)$ . . . . .	63
14. Reduction to the case where $\mathfrak{g}$ is solvable and of dimension 5 . . . . .	66
15. The existence of a 3-dimensional abelian ideal in $\mathfrak{g}$ suffices . . . . .	69
16. The final steps . . . . .	77
References . . . . .	80

## 1. Introduction

The main topic of this paper is the study of real-analytic CR-manifolds  $M$  with everywhere degenerate Levi form. In particular, for homogeneous manifolds of this type, we develop methods for the computation of the Lie algebras  $\mathfrak{ho}(M, a)$  of infinitesimal CR-transformations at every  $a \in M$ . We also classify, up to local CR-equivalence, all locally homogeneous degenerate CR-manifolds in dimension 5.