ON THE BOUNDARY BEHAVIOUR OF ELLIPTIC MODULAR FUNCTIONS.

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I.

Introduction.

1. Let $\tau = x + iy$ be a complex variable, and $q = e^{i\pi\tau}$; let us write, following the notation of TANNERY and MOLK[1],

$$\vartheta_2(0 \mid \tau) = 2q^{\frac{1}{4}} + 2q^{\frac{9}{4}} + 2q^{\frac{25}{4}} + \cdots$$

$$\vartheta_3(0 \mid \tau) = 1 + 2q + 2q^4 + 2q^9 + \cdots$$

$$\vartheta_4(0 \mid \tau) = 1 - 2q + 2q^4 - 2q^9 + \cdots$$

These series are convergent when $y > 0$ and represent functions which are analytic in the half-plane $y > 0$ and which cannot be continued across the x-axis. The behaviour of these functions, when τ tends to a real number ξ by moving along the straight line $x = \xi$, has many interesting features. When ξ is rational, the behaviour is fairly simple and can be obtained either directly[2], or by effecting on τ a suitable linear transformation

[1] TANNERY and MOLK, Éléments de la théorie des Fonctions Elliptiques, Vol. II (1896), p. 257. We shall refer to this book (Vol. II) as F.E.

[2] Cf. HARDY, On the representation of a number as the sum of any number of squares, and in particular of five, *Transactions of the American Mathematical Society,* Vol. XXI (1920) pp. 255—284 (p. 259). Though the direct method gives the result in many cases without necessitating an appeal to the transformation theory, the latter has the advantage of being applicable to all elliptic modular functions, including those for which a direct method is not available. See HARDY and RAMANUJAN, Asymptotic formulae in combinatory analysis, *Proceedings of the London Mathematical Society,* (Ser. 2) Vol. 17 (1918) pp. 75—115 (pp. 93, 94).