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Inclusion relations among methods of summability compounded from given matrix methods

By RALPH PALMER AGNEW

1. Introduction. It is our object to clarify and generalize some theorems on inclusion among matrix methods of summability given by RUDBERG [1944], and to give applications involving the Cesàro, Abel, Euler, Borel, binary, and other methods. While RUDBERG gives no references, we observe that some fundamental ideas underlying the paper of RUDBERG and this one were used by HARDY and CHAPMAN [1911], JACOBSTHAL [1920] and KNOPP [1920]. Other references appear later.

For each $r = 1, 2, 3, \ldots$, let $A(r)$ be a triangular matrix of elements $a_{nk}(r)$ such that

$$a_{nk}(r) \geqq 0,\ a_{nn} > 0, \qquad 0 \leqq k \leqq n;\ n = 0, 1, \ldots \tag{1.01}$$

$$\lim_{n\to\infty} a_{nk}(r) = 0 \qquad k = 0, 1, 2, \ldots \tag{1.02}$$

and

$$\lim_{n\to\infty} \sum_{k=0}^{n} a_{nk}(r) = 1. \tag{1.03}$$

Then, for each r, $A(r)$ determines a regular Silverman-Toeplitz transformation

$$\sigma_n(r) = \sum_{k=0}^{n} a_{nk}(r)\, s_k \tag{1.1}$$

by means of which a given sequence s_n is evaluable to s if $\sigma_n(r) \to s$ as $n \to \infty$. Our terminology agrees with that of HARDY [1949].

Let the elements of a given sequence $s_0, s_1, s_2, \ldots$ be denoted by $s_0(0)$, $s_1(0), s_2(0), \ldots$. Let $s_0(1), s_1(1), s_2(1), \ldots$ denote the $A(1)$ transform of $s_0(0)$, $s_1(0), s_2(0), \ldots$; let $s_0(2), s_1(2), s_2(2), \ldots$ denote the A_2 transform of $s_0(1), s_1(1)$, $s_2(1), \ldots$; and so on. Then, for each $r = 1, 2, 3, \ldots$

$$s_n(r) = \sum_{k=0}^{\infty} a_{nk}(r)\, s_k(r-1) \qquad n = 0, 1, 2, \ldots. \tag{1.2}$$

The elements of these sequences form the double sequence