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Retraction and extension of mappings of metric and non-metric spaces

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Introduction

1. The two kinds of topological spaces that are called absolute retracts and absolute neighborhood retracts, were originally defined by BORSUK ([5], [6]) for compact metric spaces. Later on these concepts were extended to several other classes of spaces.

A closed subset X of a space Z is called a retract of Z if there is a mapping $r: Z \to X$ such that $r(x) = x$ for each $x \in X$. The mapping r itself is called a retraction of Z onto X. By an absolute retract we mean a space X, such that whenever X is imbedded as a closed subset of a space Z, X is a retract of Z. However, if this definition is to have a meaning, we have to determine which spaces Z are allowed. There is, for instance, an example (example 17.7) of a separable metric space X, which is a retract of any separable metric space in which it is imbedded as a closed subset, but which can be imbedded as a closed subset of a normal space Z in such a way that it is not a retract of Z.

A closed subset X of a space Z is called a neighborhood retract of Z, if there is an open set O in Z, such that $X \subset O$, and a retraction $r: O \to X$. The mapping r itself is called a neighborhood retraction. By an absolute neighborhood retract we mean a space X such that whenever X is imbedded as a closed subset of a space Z, X is a neighborhood retract of Z. Again we must know which spaces Z are allowed. In order to give a simple example let X be the Hausdorff space consisting of only two points. This is a neighborhood retract of any Hausdorff space Z in which it is imbedded. However, it is not necessarily a neighborhood retract when imbedded in a T_1-space.

Thus when changing the class of spaces from which Z shall be taken, we get different concepts absolute retract and absolute neighborhood retract. The purpose of this paper is to study the properties of these concepts and the relationships between them.

We will mainly concentrate on some special classes of spaces. These classes are listed in § 2. In §§ 3–7 we have gathered together some facts about these spaces that will be useful in the sequel.

In § 2 we also define two other kinds of spaces, called extension spaces and neighborhood extension spaces. We will show in §§ 8–11 that they are closely related to absolute retracts and absolute neighborhood retracts. We study in § 12 our four concepts for contractible spaces.