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# On approximation of continuous and of analytic functions

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## 1) General survey

Let $\{\xi_{\nu,n}\}$ denote a system of points in the interval (0, 1) with the following properties

$$n = 1, 2, 3, \ldots$$
$$\nu = 0, 1, \ldots, (n-1), n.$$
$$\xi_{\nu,n} > \xi_{\mu,n} \quad \text{if} \quad \nu > \mu.$$

With every point $\xi_{\nu,n}$ we associate a real function $\psi_{\nu,n}(x)$, defined for $0 \leqq x \leqq 1$.

A system of the above-mentioned type will be said to solve the approximation problem, if for every continuous function $f(x)$

$$A_n(f) = \sum_{\nu=0}^{n} f(\xi_{\nu,n})\, \psi_{\nu,n}(x)$$

tends to $f(x)$ when $n$ tends to infinity, uniformly for $0 \leqq x \leqq 1$.

In this paper we are going to treat the case when the approximation functions $\psi_{\nu,n}(x)$ are non-negative. We begin in section 2 by stating the necessary and sufficient conditions of a system $\{\xi_{\nu,n}\}$ of points. We proceed in section 3 by stating the necessary and sufficient conditions of a system $\{\xi_{\nu,n};\, \psi_{\nu,n}\}$ of points and functions, which solves the approximation problem. Then in section 4 we apply the obtained results on a special system and finally, in section 5, we study the convergence for complex values of $x$ for this same system.

## 2) Necessary and sufficient conditions of $\{\xi_{\nu,n}\}$.

We shall prove that the conditions

$$\left.\begin{aligned} &\xi_{0,n} \to 0 \\ &\xi_{n,n} \to 1 \\ &\max_{\nu} \{\xi_{\nu+1,n} - \xi_{\nu,n}\} \to 0 \end{aligned}\right\}$$

when $n \to \infty$ are necessary and sufficient for $\{\xi_{\nu,n}\}$ in the following meaning.