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On an inequality concerning the integrals of moduli of regular analytic functions

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With 1 figure in the text

1. Let $f(z)$ be an analytic function, regular in a convex domain D and on its boundary C. Let L be a rectifiable curve in D. Our problem is to estimate

$$\int_L |f(z)\,dz|$$

by means of the integral

$$\int_C |f(t)\,dt|.$$

Professor F. Carlson (1) has shown that in the case D being a circle, then

$$\int_L |f(z)\,dz| \leq \frac{1}{\pi}\int_C V(t)\,|f(t)\,dt$$

where $V(t)$ is the upper limit of the sum of the angles at which the elements of L are seen from a point t on C. He has called my attention to the possibility of solving the problem for convex domains by the same method as the one he uses for a circle.

Gabriel, in a first work (2), has treated the problem for a circle and in a second work (3) for convex regions.

2. Let L be a rectilinear segment in D. We may suppose that L is parallel to the real axis. Let $F(\zeta, z)$ be a function that for each $z \in D$ is an analytic function of ζ, regular in D and continuous on C. Then, by Cauchy's theorem, we have

$$f(z) = \frac{1}{2\pi i}\int_C \frac{F(t,z)}{F(z,z)}\frac{f(t)}{t-z}\,dt.$$

Hence

$$|f(z)| \leq \frac{1}{2\pi}\int_C \left|\frac{F(t,z)}{F(z,z)}\,\frac{1}{t-z}\right| |f(t)\,dt|$$

and