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# On null-sets for continuous analytic functions

By LENNART CARLESON

**1.** Let $E$ be a compact set with a connected complement $\Omega$. If $\Gamma$ is the class of functions $f(z)$ which are analytic in $\Omega$ and have a certain property $P$, then a set $E$ is said to be a "null-set" with respect to $\Gamma$, if this class consists entirely of constants. An investigation of these null-sets for certain properties $P$ has recently been published by AHLFORS and BEURLING [1]. For example they consider the PAINLEVÉ problem where $P$ is boundedness. In this paper $P$ is a continuity property, and our aim is to give metrical conditions on the corresponding null-sets.

We denote by $L_\alpha(E)$ and $C_\alpha(E)$ HAUSDORFF measure and capacity of order $\alpha$, $0 < \alpha < 2$; for their definitions we refer to [2] A function $f(z)$ (not necessarily single valued) is said to belong to Lip $\alpha$, $0 < \alpha < 1$, if for every circular arc $\gamma$ of length $|\gamma| < 1$ and for every branch of $f(z)$,

$$\left|\int_\gamma f'(z)\,dz\right| \leq M\,|\gamma|^\alpha,$$

where $M$ is a constant independent of $\gamma$.

**2.** Our first theorem is concerned with multiple valued functions $f(z)$.

**Theorem:** *Let $\Gamma$ be the class of functions belonging to* Lip $\alpha$ *and having single valued real part. Then $E$ is a null-set if and only if*

$$L_\alpha(E) = 0.$$

If $L_\alpha(E) > 0$, then there exists a real, completely additive set function $\mu$ vanishing outside $E$ such that

(a) $\mu(E) = 0$,

(b) $\int_E |d\mu| = 1$,

(c) $|\mu(C)| \leq M\,r^\alpha$ for every circle $C$ of radius $r$.

The function

$$f(z) = \int_E \log(z - \zeta)\,d\mu(\zeta)$$