Invariant sets under iteration of rational functions

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Introduction

The theory of the iteration of a rational function $R(z)$ developed by Fatou [5-6] and Julia [9] treats the sequence of iterates $\{R_n(z)\}$ defined by

$$R_0(z) = z, \quad R_1(z) = R(z), \quad R_{n+1}(z) = R(R_n(z)), \quad n = 0, 1, 2, \ldots$$

A fundamental role is played here by the set $F$ of those points of the complex plane