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A set of uniqueness for functions, analytic and bounded in the unit disc

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1. Introduction

The purpose of this note is to establish a uniqueness theorem, similar to the well-known result of F. and M. Riesz. Before we state the theorem, let us introduce some notation.

Throughout this note let $\mathfrak{J}$ be the class of all functions, analytic and bounded in the open unit disc C. We will also consider the subclass $\mathfrak{J}_0 \subset \mathfrak{J}$ of functions, with only a finite number of zeros in C.

If ζ is a point on the boundary of C (henceforth denoted by ∂C) and α is a real number, $0 \leqslant \alpha < 1$, let $S(\zeta, \alpha)$ denote the Stolz domain with vertex $\zeta \in \partial C$ and angle $\arcsin \alpha$; i.e.

$$S(\zeta, \alpha) = \{z \mid |z| < 1, \ |z - \zeta| < \sqrt{1-\alpha^2}, \ |\arg(1 - \bar{\zeta} z)| \leqslant \arcsin \alpha\}.$$

Moreover, if $\zeta \in \partial C$ and φ is a function, defined on C, such that

$$\lim_{\substack{z \to \zeta \\ z \in S(\zeta, \alpha)}} \varphi(z) = A \quad \text{for all} \quad \alpha, \ 0 \leqslant \alpha < 1,$$

we write

$$\lim_{z \to \zeta}{}^{S} \varphi(z) = A \quad \text{or} \quad \varphi(z) \xrightarrow{S} A \quad \text{as} \quad z \to \zeta.$$

We will use the first notation exclusively when A is a (proper) complex number, while the second notation will be used not only when A is a proper complex number but also in the case of a real-valued function φ and $A = \pm\infty$.

For $f, g \in \mathfrak{J}$ consider the set

$$D_S(f, g) = \{\zeta \mid \zeta \in \partial C, \ \lim_{z \to \zeta}{}^{S} f^{(k)}(z) = \lim_{z \to \zeta}{}^{S} g^{(k)}(z), \ k = 0, 1, 2, \ldots\}.$$

An immediate consequence of F. and M. Riesz's theorem ([2], p. 209) is the following result:

If $D_S(f, g)$ has positive Lebesgue measure, then $f = g$.

The main result to be proved in this note can be stated as follows: