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Some problems related to iterative methods in conformal mapping

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Introduction

A. The conformal mapping problem for domains of connectivity greater than one has been attacked in several ways. The desired mapping function can sometimes be found as the solution of an extremal problem or of an integral equation or its existence may in certain cases be proved by means of the method of continuity. Another method, sometimes called *the function-theoretical iteration process*, is to express the mapping function $F(z)$ as a composition of functions $\{f_n\}_1^\infty$:

$$F_n(z)=f_n(f_{n-1}(\dots(f_1(z))\dots));\qquad F(z)=\lim_{n\to\infty}F_n(z),\tag{A1}$$

where the f_n (determined in some way, e.g. see Hübner below) are meromorphic and univalent in certain *simply* connected domains. An advantage of this method is that it connects the theoretical and the constructive questions about the mapping.

Hübner ([10] pp. 43–55) has constructed a process—*the general iteration process*—by means of which every function $F(z)$ conformally mapping one domain onto a domain with analytic boundary can be expressed according to (A1). Thus, theoretically several of the well-known canonical mappings can be expressed in this way. However, the determination of f_n, $n=1, 2, \dots$, as a rule requires knowledge of $F(z)$ itself and thus the process from a constructive point of view has little interest. But there exist exceptions, namely the circular ring mapping and the mapping onto the lemniscate domain, studied earlier by Walsh, Grunsky and Landau.

A detailed account of the problems referred to in this section and the following is found in [3] pp. 208–240.

B. A straightforward attempt to use the function-theoretical iteration process is sketched below. For brevity we call it *the iterative process*. Here the determination of the functions f_n causes no trouble but on the other hand the convergence question is more intricate.

Let D be a domain of connectivity $k\geq 2$ on the z-sphere with the continua $C_\nu^{(0)}$, $\nu=1, 2, \dots, k$, as boundary components. Let $D^{(n)}=F_n(D)$ (F_n to be determined later) have boundary components $C_\nu^{(n)}$ corresponding to $C_\nu^{(0)}$, $\nu=1, 2, \dots, k$. It is required to find $F(z)$, eventually restricted by some normalization conditions, conformally mapping D onto a domain Ω so that $C_\nu^{(0)}$ corresponds to L_ν, $\nu=1, 2, \dots, k$. Here L_ν