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# On a class of Diophantine equations of the second degree in imaginary quadratic fields

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## Introduction

The problem of solving the Diophantine equation

$$u^2 - D v^2 = N, \tag{1}$$

where $D$ and $N$ are rational integers and where $D$ is not a perfect square, in rational integers has usually been treated by using either the theory of quadratic forms or the theory of quadratic fields. T. Nagell [1], [2], [3], [4][1] has shown, however, how it is possible to determine all the solutions of (1) completely elementarily and without using either of the theories mentioned above.

The purpose of this paper is to show that Nagell's method can also be used to determine the solutions in integers belonging to an imaginary quadratic number field, of equation (1), when $D$ and $N$ are integers in the field considered, and $D$ is not a perfect square in that field.

I treat in § 1—§ 3 the equation

$$x^2 - \delta y^2 = \pm 1, \tag{2}$$

and in § 5 of this paper I will show how the theory developed here can be used for studying equation (1).

In § 4 we make a closer investigation of a special case of (2) and connect the equation with the units in certain quartic fields.

## § 1. A lemma and its application

The theory of the Diophantine equation $x^2 - \delta y^2 = 1$, can easily be developed starting from the following

**Lemma 1:** *Let $\alpha$ be any complex number which does not belong to the field $K(\sqrt{-m})$, where $m$ is a squarefree natural number and where $\sqrt{-m}$ is taken to be $i\sqrt{m}$. Then the Diophantine inequality*

$$|x - \alpha y|^2 < \frac{(m+1)^2}{N(y)}, \tag{3}$$

[1] Figures in [ ] refer to the Bibliography at the end of this paper.