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Direct sum decompositions in Grothendieck categories

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Throughout this paper, $\mathcal{A}$ will denote a Grothendieck category, i.e. an abelian category with generators and exact direct limits. Our main theorem gives a sufficient condition for an object to decompose into a direct sum of indecomposable objects. This theorem will then be applied to obtain decompositions of injective objects in locally noetherian categories and of projective modules over perfect rings. Some applications will also be given to relative splitting problems, i.e. splitting by $\mathcal{E}$-proper subobjects where $\mathcal{E}$ is a proper class in the sense of relative homological algebra.

In a preliminary version of this paper (cited in [12]), $\mathcal{A}$ was assumed to be locally finitely generated. I am grateful to J.-E. Roos for pointing out that the results may be extended to AB 6 categories.

1. Main theorem

For the validity of the subsequent decomposition theorems it turns out to be essential that $\mathcal{A}$ should satisfy some condition of local finiteness. The following two axioms will be used for this purpose:

AB 6: Every object M is a sum of subobjects which are finitely generated relative to M.

AB 6 (ess): Every object $M \neq 0$ contains a subobject $\neq 0$ which is finitely generated relative to M.

Here a subobject L of M is called *finitely generated relative to M* if whenever $M = \sum M_i$ for a directed family $(M_i)_I$ of subobjects of M, there is an $i \in I$ such that $L \subset M_i$. The axiom AB 6 was first introduced in [6] in the following form:

For every M and every family $(M_j)_J$ of directed families of subobjects $(M_{j\alpha})_{A_j}$ of M, the canonical morphism

$$\varphi: \sum_{\alpha(j) \in \pi A_j} \left(\bigcap_{j \in J} M_{j\alpha}\right) \to \bigcap_{j \in J} \left(\sum_{\alpha \in A_j} M_{j\alpha}\right)$$

is an isomorphism.