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On bounded analytic functions and closure problems

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Introduction

1. Let us denote by H^p, $p \geq 1$, the space of functions $f(z)$ holomorphic in $|z| < 1$ and such that

$$N_p(f) = \lim_{r \to 1} \left\{ \frac{1}{2\pi} \int_0^{2\pi} |f(re^{i\theta})|^p \, d\theta \right\}^{\frac{1}{p}} = \left\{ \frac{1}{2\pi} \int_0^{2\pi} |f(e^{i\theta})|^p \, d\theta \right\}^{\frac{1}{p}} < \infty,$$

where $f(e^{i\theta}) = \lim_{r \to 1} f(re^{i\theta})$ a. e. It is obvious that H^p is a Banach space under the norm N_p. If we combine the wellknown representation of a linear functional on $L^p(0, 2\pi)$ with a theorem of M. Riesz on conjugate functions, we find that the general linear functional on H^p, $p > 1$, has the form

$$L(f) = \int_0^{2\pi} f(e^{i\theta}) \overline{g(e^{i\theta})} \, d\theta, \quad g \in H^q, \quad p^{-1} + q^{-1} = 1. \tag{1}$$

The simple structure of the general linear functional on H^p is the key to a great number of results for these spaces.

The "limit space" as $p \to \infty$ is the space B of bounded analytic functions in $|z| < 1$ with the uniform norm

$$\|f\| = \sup_{|z| < 1} |f(z)|. \tag{2}$$

Although this space has a simpler function-theoretic nature than H^p, its theory as a Banachspace is extremely complicated. This fact depends to a great extent on the absence of a simple representation for linear functionals. On the other hand, B is not only a Banach space, but also a Banach algebra.

If one seeks results for B which for H^p depend on the formula (1), the following question should be asked: how shall we weaken the norm (2) in B in order to ensure that the functionals have a representation of type (1)? In the first section we shall treat this problem by introducing certain weight functions. The method will also be used to find a function-theoretic correspondance to weak convergence on a finite interval.