Communicated 24 October 1951 by F. CARLSON and T. NAGELL

On the Diophantine equation $u^2 - Dv^2 = \pm 4N$

By BENGT STOLT

Part II

§ 1. Introduction.

Consider the Diophantine equation

$$u^2 - Dv^2 = \pm 4N, \tag{1}$$

where D and N are integers and D is not a perfect square. In Part I of this investigation[1] it was shown that it is possible to determine all the solutions of (1) by elementary methods[2].

Suppose that (1) is solvable, and let u and v be two integers satisfying (1). Then $\frac{u + v\sqrt{D}}{2}$ is called a *solution* of (1). If $\frac{x + y\sqrt{D}}{2}$ is a solution of the Diophantine equation

$$x^2 - Dy^2 = 4, \tag{2}$$

the number

$$\frac{u + v\sqrt{D}}{2} \cdot \frac{x + y\sqrt{D}}{2} = \frac{u_1 + v_1\sqrt{D}}{2}$$

is also a solution of (1). This solution is said to be *associated* with the solution $\frac{u + v\sqrt{D}}{2}$. The set of all solutions associated with each other forms a *class of solutions* of (1).

A necessary and sufficient condition for the two solutions $\frac{u + v\sqrt{D}}{2}$, $\frac{u' + v'\sqrt{D}}{2}$ to belong to the same class is that the number

$$\frac{vu' - u'v}{2N}$$

be an integer.

[1] See [1].

[2] These methods were developed by T. NAGELL, who used them for determining all the solutions of the Diophantine equation

$$u^2 - Dv^2 = \pm N.$$

Nagell also proposed the notions used in this section. See [2], [3], [4], [5].