

Convexity and norm in topological groups

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With 2 figures in the text

§ 1. Introduction and preliminaries

1.1. This work originated in a simple observation on the behaviour of sets of points in the euclidean plane when a certain operation of taking powers is performed on them. Let A and B be two sets in the plane. We define $A+B$ to be the set of all points which can be expressed in the form $a+b$ where $a \in A$, $b \in B$ and $a+b$ is the usual vectorial addition of points. The expression $A+A+\dots A$ will be denoted by A^n if the repeated sum contains n terms, and we shall call A^n the n :th power of A . The reason for choosing the symbol A^n instead of nA is that for real λ the symbol λA has already a universally accepted meaning.

The observation mentioned above may be formulated as follows: The higher the power is, the higher is the degree of convexity. (At present we shall make no attempt to give this statement a precise meaning.) An inspection of Fig. 1 and 2 may make the meaning of this vague statement more clear.

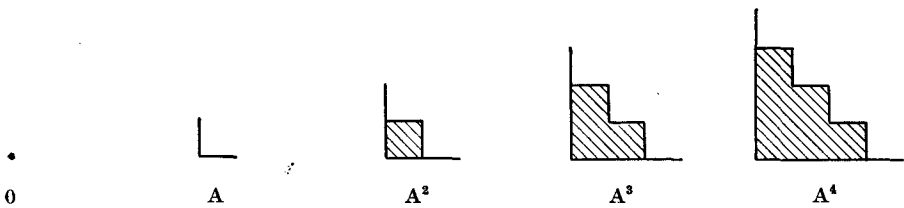


Fig. 1.

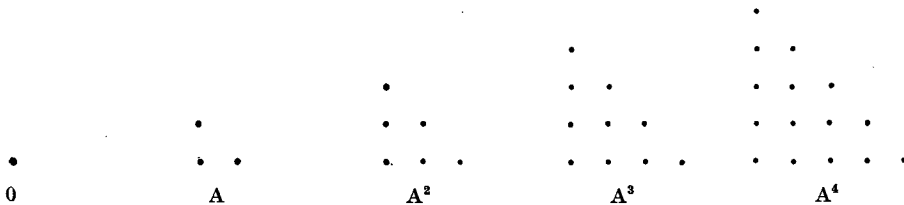


Fig. 2.