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Stochastic processes and statistical inference

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Introduction

The purpose of this thesis is, partly to show the possibility of applying statistical concepts and methods of inference to stochastic processes, and partly to obtain practically working methods of this kind by studying special cases of inference.

Time-series have been subjected to statistical treatment in a more or less systematical way for a very long time, but unlike the case of finite dimensional samples, there exists no unified theory. The extensive literature on stochastic processes has but rarely touched upon questions of inference. On the other hand, the attempts to treat time-series data do not seem to have been much influenced by the theory of stochastic processes. This is specially the case when considering a continuous time-parameter, which will be our main interest in the following chapters. The treatment of the problem in the present dissertation is based on the general idea outlined in CRAMÉR: Mathematical methods of statistics — to base statistical methods on the mathematical theory of probability.

In the first two chapters we shall give a short survey of some fundamental facts about stochastic processes and statistical inference. The third and fourth chapters will deal with the problem of testing hypotheses and the fifth with estimation. Finally in the sixth chapter we shall show very shortly that prognosis and filtering of time-series are questions similar to testing and estimation and can be treated on analogous lines.

Some topics in the theory of stochastic processes

1.1. Measure of probability. Let us consider an abstract space Ω with the following properties. The points in Ω are denoted by ω. In Ω is defined a Borelfield of sets containing also Ω. On this Borelfield there is defined a completely additive, non-negative setfunction P for which $P(\Omega) = 1$. Then P is said to be a probability-measure on Ω. It is sometimes convenient to close the measure by defining every set, which can be enclosed by a set (belonging to the Borelfield) of measure zero, as measurable with measure zero.