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Propagation of analyticity of solutions of partial differential equations with constant coefficients

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1. Introduction

In [6] L. Hörmander discusses the following problem: Given an open set Ω in $\mathbf{R}^n$, two relatively closed subset X_1, X_2 of Ω and a partial differential operator $P(D)$ with constant coefficients, which is then the smallest set $X \subset \Omega$ such that

$$\text{s.s.}U \subset X_1,\ \text{s.s.}P(D)\,U \subset X_2 \Rightarrow \text{s.s.}U \subset X,\ U \in \mathcal{D}'\,(\Omega). \tag{1.1}$$

Here s.s.U denotes the singular support of U, i.e. the smallest, relatively closed subset of Ω such that U is an infinitely differentiable function in $\Omega \setminus \text{s.s.}U$.

In some cases the problem has been solved completely. Let $P_m(D)$ be the principal part of $P(D)$. $P(D)$ is said to be of principal type if grad $P_m(\xi) \neq 0$ when $\xi \in \dot{\mathbf{R}}^n = \mathbf{R}^n \setminus \{0\}$ and $P(D)$ is called real if it has real coefficients. If $P(D)$ is of principal type and $P_m(D)$ is real then a line with direction grad $P_m(\xi)$, for some $\xi \in \dot{\mathbf{R}}^n$ satisfying $P_m(\xi) = 0$, is called bicharacteristic. The following results are known [4, 6, 11]:

Theorem 1.1. *Suppose that $P(D)$ is of principal type and that $P_m(D)$ is real. If l is a bicharacteristic line and I is any closed interval (finite or infinite) contained in l, then there is a distribution F such that* s.s.$F = I$ *and $P(D)F$ is infinitely differentiable except at the (finite) endpoints of I.*

Theorem 1.2. *Let $P(D)$ be as in Theorem 1.1 and let H be a closed cone in $\mathbf{R}^n$ containing one half ray of every bicharacteristic line for $P(D)$ through the origin. Then there is a fundamental solution E of $P(D)$ with* s.s.$E \subset H$.

These two thereoms together easily give:

Theorem 1.3. *If $P(D)$ is of principal type and $P_m(D)$ is real then* (1.1) *is valid if and only if*

$$X \supset X_1 \cap X_2 \text{ and, for every bicharacteristice line } l\text{, the set } X \text{ contains any component } I \text{ of } l \cap (\Omega \setminus X_1 \cap X_2) \text{ such that } I \subset X_1. \tag{1.2}$$

The main purpose of this paper is to prove that Theorem 1.3 is valid also when infinite differentiability is replaced by real analyticity. To do so, we will extend Theorems 1.1 and 1.2 to the anlytic case. Theorem 1.1 extends word for word, but in the theorem corresponding to Theorem 1.2 it seems necessary to strengthen the as-