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Analytic structures in the maximal ideal space of a uniform algebra

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Introduction

Let A be a uniform algebra with its maximal ideal space M_A and its Shilov boundary S_A. We say that M_A has a (one-dimensional) analytic structure at a point $x_0 \in M_A \setminus S_A$ if the following condition holds. There is an open neighborhood W of x_0 in M_A and some $f \in A$ such that $W \setminus \{x_0\} = V_1 \cup \ldots \cup V_n$, where V_i are disjoint open subsets of M_A each mapped homeomorphically by f onto the set $D \setminus \{0\}$ in C^1. Here D is the open unit disc and $f(x_0)=0$. The positive integer n above is called the branch-order of x_0.

If M_A has an analytic structure at the point x_0 as above, then J. Wermer has proved that if $g \in Z$ and if we define $g_i(z)=g(x_i(z))$ on $D \setminus \{0\}$, where $x_i(z)$ is the point in V_i for which $f(x_i(z))=z$ while $g_i(0)=g(x_0)$, then $g_1 \ldots g_n$ are analytic functions in D.

Conditions which guarantee that subsets of $M_A \setminus S_A$ have an analytic structure have originally been studied by J. Wermer in [5–6]. In Section 1 of this paper we prove some results which originally were obtained by Wermer under certain regularity conditions. The core of this section is the proof of Theorem 1.7. and in the final part we discuss some consequences of this result.

Section 1

Firstly we introduce some notations and collect some wellknown facts about uniform algebras. If X is a compact space and if $f \in C(X)$ we put $|f|_X = \sup\{|f(x)| : x \in X\}$. If W is a subset of X then ∂W denotes its topological boundary. If A is a uniform algebra and if F is closed subset of M_A we put $\mathrm{Hull}_A(F)=\{x \in M_A : |f(x)| \leqslant |f|_F$ for all f in $A\}$. We also introduce the uniform algebra $A(F)=\{g \in C(F) : \exists (f_n)$ in A with $\lim |f_n - g|_F = 0\}$. Here we know that $M_{A(F)}$ can be identified with $\mathrm{Hull}_A(F)$.

If A is a uniform algebra and if $f \in A$ we define the fibers $\pi_f^{-1}(z)=\{x \in M_A : f(x)=z\}$ for each $z \in C^1$. We recall the wellknown result below.

Lemma 1.1. *Let A be a uniform algebra and let $f \in A$. Suppose that U is an open component of the set $C^1 \setminus f(S_A)$. Then two cases are possible, either $U \cap f(M_A)$ is empty or else $U \subset f(M_A)$.*

Next we introduce a concept which appears in [2, p. 525].