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On the joint distribution of crossings of high multiple levels by a stationary Gaussian process

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1. Introduction

Let $\{\xi(t), -\infty<t<\infty\}$ be a real stationary Gaussian process with zero mean function and having continuous sample paths with probability one. Denote the covariance function by r (taking $r(0)=1$ for convenience), and the corresponding spectral distribution function by F. Let μ be the expected number of upcrossings of the level u by $\xi(t)$ in a t-interval of length 1.

Under certain conditions, H. Cramér [2, pp. 258 ff.] has shown that the number of upcrossings by $\xi(t)$ during a t-interval of length T of a single level tending to infinity is asymptotically Poisson distributed with parameter τ, provided T is chosen tending to infinity according to $T=\tau/\mu$. Cramér's conditions for validity have been weakened, in slightly different directions, by Belayev [1] and the author [3].

In this paper we show that a multivariate Poisson distribution is obtained in the analogous situation for upcrossings of multiple levels. The conditions for validity are the weakened ones of [3]. For the following precise statement of the result we need some notation. Let $0<p_l\leqslant p_{l-1}\leqslant\ldots\leqslant p_1\leqslant p_0=1$, and consider the levels $u, u-(\ln p_1)/u, \ldots, u-(\ln p_l)/u$, and the t-interval $(0, T)$ where $T=\tau/\mu$, $\tau>0$.

Let $N_0, N_1, \ldots, N_l$ be the numbers of upcrossings by $\xi(t)$ during time T of these $l+1$ levels in the order listed.

Theorem 1.1. *If the stationary Gaussian process $\xi(t)$ satisfies*

(1) $\lambda_2=-r''(0)$ *exists and* $\int_0^\delta (\lambda_2+r''(t))/t\,dt<\infty$, *for some* $\delta>0$,

$$\textit{or equivalently, } \int_0^\infty \log\,(1+\lambda)\,\lambda^2 dF(\lambda)<\infty, \textit{ and}$$

(2) $r(t)=0(t^{-\alpha})$ *as* $t\to\infty$ *for some* $\alpha>0$,

then