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Analyticity of fundamental solutions

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Introduction

Trèves and Zerner [1] have studied analyticity domains of fundamental solutions of linear partial differential operators with constant coefficients. They formulate a general criterion that ensures the existence of a fundamental solution which is analytic in the complement of a certain algebraic conoid and this criterion shows that an operator with real principal part and simple real characteristics has a fundamental solution which is analytic in the complement of the bicharacteristic conoid. They also show that a semielliptic operator has a fundamental solution which is analytic outside a certain linear subspace of R^n.

In section 1 of this paper we give a criterion (announced in [2]) different from that of [1], geared to the classical method of constructing fundamental solutions by integrating over suitable chains in complex space where the Fourier kernel is small and the characteristic polynomial does not vanish. In section 2 this criterion is applied to hypoelliptic, in particular semielliptic, operators and to operators with real principal part and simple real characteristics. Trèves and Zerner remark (l.c. p. 156) that their method does not seem to work in the case of complex coefficients. In section 3 we give a simple result for such operators. Finally, we generalize this result to products of operators. In an appendix we have gathered some simple facts, used in section 3, concerning convergence of distributions.

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1. Vectorfields and fundamental solutions

Points in R^n will usually be denoted by x, y or ξ, η and when $z = x + iy \in \mathbf{C}^n$ we write $\operatorname{Re} z = x$, $\operatorname{Im} z = y$ and $\bar{z} = x - iy$. On $\mathbf{C}^n$ we use the duality $z \cdot \zeta = z\zeta = z_1\zeta_1 + \ldots + z_n\zeta_n$ and the norm $|z| = (z \cdot \bar{z})^{\frac{1}{2}}$. When $P(\xi)$ is a polynomial let $P(D)$ be the associated differential operator, where $D_k = \partial / i\partial x_k$ and $D = (D_1, \ldots, D_n)$. Let $P_k(\xi)$ be the part of P of homogeneity k so that $P = P_m + P_{m-1} + \ldots$, where m is the degree of P and hence $P_m(D)$ the principal part of $P(D)$.

Given a differential operator $P(D)$, we denote by $V = V(P)$ the family of vectorfields

$$R^n \ni \xi \to w(\xi) \in \mathbf{C}^n$$