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The Stone-Weierstrass theorem in Lipschitz algebras

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1. Introduction

A normed function algebra is said to have the Stone-Weierstrass property if every subalgebra which separates points and contains constant functions is dense in the algebra. The purpose of this paper is to investigate this property in certain algebras of real-valued functions with norm greater than the sup norm.

Let X be a compact metric space, connected or not, with metric $d(x,y)$. Let Lip (X,d^α), $0<\alpha\leqslant 1$, be the Banach algebra of all real-valued functions f on X such that

$$\|f\| = \operatorname{Max}\{\sup_{x\in X}|f(x)|, \sup_{x,y\in X}|f(x)-f(y)|/d(x,y)^\alpha\}<\infty$$

and let lip (X,d^α) be the subset of all f in Lip (X,d^α) with the property that

$$\sup\{|f(x)-f(y)|/d(x,y)^\alpha;\ x,y\in X, d(x,y)\leqslant\delta\}\to 0 \quad \text{as} \quad \delta\to 0.$$

If $0<\alpha<1$, lip (X,d^α) always contains plenty of functions, and it is a point-separating closed proper subalgebra of Lip (X,d^α). See [3] where these algebras are studied in detail.

It is natural to ask if lip (X,d^α) has the Stone-Weierstrass property (which, obviously, Lip (X,d^α) does not have). However, in [3], p. 249, reference is made to an unpublished example by Katznelson of a point-separating subalgebra of lip (X,d^α) which is not dense in lip (X,d^α). In the first part of this paper we give a necessary and sufficient condition, in terms of local properties of the functions, for a point-separating subalgebra of lip (X,d^α) to be dense (Theorem 1, Corollaries 1 and 2).

In the second part we consider algebras of periodic functions on the real line. For $0<\alpha<1$ we denote by Λ_α the algebra of all continuous real-valued functions with period 2π such that

$$\|f\| = \operatorname{Max}\left\{\sup_x|f(x)|, \sup_{t>0} t^{-\alpha}\int_{-\pi}^{\pi}|f(x+t)-f(x)|\,dx\right\}<\infty$$

and by λ_α the closed subalgebra of functions such that

$$\lim_{t\to 0} t^{-\alpha}\int_{-\pi}^{\pi}|f(x+t)-f(x)|\,dx = 0.$$

For $0<\alpha<2$ we denote by B_α the algebra of all continuous real-valued functions with period 2π such that