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A note on asymptotic normality of sums of higher-dimensionally indexed random variables

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1. Summary and notation

We shall consider asymptotic normality of sums of random variables when the domain of the summation index is a subset of the lattice points in some higher-dimensional space. Our main aim is to point out that the idea used by the author in [6] to treat asymptotic normality of sums of "one-dimensionally" indexed random variables can easily be adapted to the case of higher-dimensionally indexed random variables.

The course of the paper is as follows. In section 2 we state a result about asymptotic normality, which is equivalent to the author's theorem A in [6]. In the following two sections we illustrate the general idea by considering two particular cases. In section 3 we consider general m-dependent random variables, and section 4 is devoted to U-statistics (see [1]) in the case $\zeta_1=0$ (according to Hoeffding's notation [1]).

We use the following notation and conventions throughout the paper. E denotes expectation and σ^2 variance. $\mathcal{L}(X)$ stands for the law of the random variable, or vector, X. $\mathcal{B}(X_1, X_2, ..., X_n)$ is the σ-algebra of events generated by the random variables $X_1, X_2, ..., X_n$. $E^{\mathcal{B}}$ denotes the conditional expectation given the σ-algebra $\mathcal{B}$. We usually write E^X instead of $E^{\mathcal{B}(X)}$. $N(\mu, \sigma^2)$ denotes the normal distribution with mean μ and variance σ^2. Convergence in distribution is denoted by $\Rightarrow$. When we put a non-integer, λ, in a place where there should naturally be an integer we interprete λ as its integral part $[\lambda]$.

2. A general result about asymptotic normality

The following theorem is equivalent to theorem A in [6].

Theorem 1. *Let* $\{S_\alpha^{(n)}, 0 \leqslant \alpha \leqslant 1\}_{n=1}^{\infty}$ *be a sequence of stochastic processes on* $[0,1]$ *which satisfies* $S_0^{(n)}=0$, $n=1, 2, \ldots,$ *and the following conditions*

(C 1) *There is a function* $\chi(s)$, $0 \leqslant s \leqslant 1$, *which tends to* 0 *as* s *tends to* 0, *such that for* $0 \leqslant \beta < \alpha \leqslant 1$ *we have*

$$\overline{\lim_{n\to\infty}}\, E(S_\alpha^{(n)} - S_\beta^{(n)})^2 \leqslant \chi(\alpha-\beta), \quad 0 \leqslant \beta < \alpha \leqslant 1.$$

(C 2) *There is a function* $\varrho(\alpha)$, *continuous on* $0 \leqslant \alpha < 1$, *such that*

$$\lim_{\Delta\to+0} \frac{1}{\Delta} \overline{\lim_{n\to\infty}}\, E\,|E^{S_\alpha^{(n)}}(S_{\alpha+\Delta}^{(n)} - S_\alpha^{(n)}) - \Delta\varrho(\alpha)\, S_\alpha^{(n)}| = 0, \quad 0 \leqslant \alpha < 1.$$