ARKIV FÖR MATEMATIK Band 8 nr 1

1.68 10 1 Communicated 5 June 1968 by HARALD CRAMÉR and ULF GRENANDER

On the Hellinger integrals and interpolation of q-variate stationary stochastic processes

By HABIB SALEHI

Introduction

Let $(X_t)_{-\infty}^{\infty}$ be a q-variate continuous parameter, mean continuous, weakly stationary stochastic process (SP) with the spectral distribution measure F defined on $\mathcal{B}$ the Borel family of subsets of the real line; cf. [1]. It is known [10] that for matrix-valued measures M and N the Hellinger integral $(M, N) = \int_{-\infty}^{\infty} (dMdN^*/dF)$ ($^* =$ conjugate) may be defined in such a way that $H_{2,F}$ the space of all matrix-valued measures M for which $(M, M)_F = \int_{-\infty}^{\infty} (dMdM^*/dF)$ exist becomes a Hilbert space under the inner product $\tau(M, N)_F$ ($\tau =$ trace). The significance of these integrals when M and N are complex-valued measures and F is a non-negative real-valued measure has been pointed out by H. Cramér [2, p. 487] and U. Grenander [3, p. 207; 4, p. 195] in relation to unvariate SP's. The importance of Hellinger integrals with regard to the theory of interpolation of a q-variate weakly stationary SP with discrete time has been discussed by the author in [11]. In this paper we propose to use the Hellinger integrals and obtain similar results concerning the interpolability of a q-variate continuous parameter, mean continous, weakly stationary SP. The question of interpolability of a univariate SP with continuous time has been looked at by K. Karhunen [6]. Our results extend his work in a natural way.

Let K be any bounded measurable subset of the real line. K' will denote the complement of K in the set of the real numbers. $\mathcal{M}_K$ and $\mathcal{M}_{K'}$ will denote the (closed) subspaces spanned by $X_t, t \in K$ and $X_t, t \in K'$ respectively, i.e., $\mathcal{M}_K = \mathfrak{S}\{X_t, t \in K\}$ and $\mathcal{M}_{K'} = \mathfrak{S}\{X_t, t \in K'\}$. $\mathcal{M}_\infty$ will denote $\mathfrak{S}\{X_t, t \text{ real}\}$ and finally $\mathcal{N}_K$ will denote $\mathcal{M}_\infty \cap \mathcal{M}_{K'}^{\perp}$, where $\mathcal{M}_{K'}^{\perp}$ denotes the orthogonal complement of $\mathcal{M}_{K'}$ in a fixed Hilbert space $\mathcal{H}^q$ containing the SP $(X_t)_{-\infty}^{\infty}$.

Definition 1. We say that (a) K is interpolable with respect to (w.r.t.) $(X_t)_{-\infty}^{\infty}$ if $\mathcal{N}_K = \{0\}$.

(b) $(X_t)_{-\infty}^{\infty}$ is interpolable if each bounded measurable subset K of the real line is interpolable w.r.t. $(X_t)_{-\infty}^{\infty}$.

For each $X \in \mathcal{M}_\infty$, (X, X_t) is a continuous function on $(-\infty, \infty)$. Moreover, $(X, X_t) = 0$ iff $t \in K'$. Thus the following definition makes sense.