ARKIV FÖR MATEMATIK Band 3 nr 53

Communicated 23 April 1958 by OTTO FROSTMAN and LARS GÅRDING

On the division of distributions by polynomials

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1. Introduction

The division problem for distributions is, given a distribution T in an open set Ω in R^ν and an infinitely differentiable function φ in Ω, to find a distribution S in Ω so that

$$T = \varphi S. \tag{1.1}$$

(Cf. Schwartz [4]: Chap. V, pp. 121–126, Chap. VII, p. 154.) One may then also call S a "partie finie" of T/φ. When $\nu = 1$ the division is possible for every T if and only if φ has only isolated zeros of finite order (Schwartz [4], Chap. V, p. 123). When $\nu > 1$, however, the situation is not equally simple. It is the purpose of this paper to prove that *the division by a polynomial* (not identically zero) *is always possible.* This was conjectured by Schwartz [4], t. II, p. 154. As indicated there, this also implies that if T is a tempered distribution one can find a tempered "partie finie" S—our proof will be arranged so as to give this result directly. By applying the Fourier transformation it follows that *every partial differential equation* (for notations cf. Hörmander [1])

$$P(D)u = f \tag{1.2}$$

with constant coefficients has a tempered solution u for every tempered f. In particular, the equation has a tempered fundamental solution.

By other means, Malgrange [3] and later Hörmander [1] have proved the existence of non tempered fundamental solutions having certain local regularity properties. Such fundamental solutions were called proper by Hörmander [1], and it was also proved in that paper that there are differential equations with no fundamental solution that is both proper and tempered. This shows that the results of this paper are of a character rather different from the earlier ones of Malgrange and Hörmander, and so are the methods of proof.

Let $\mathcal{S}$ be the space of infinitely differentiable functions f in R^ν such that

$$\sup_\xi |\xi_\alpha D^\beta f(\xi)| < \infty \tag{1.3}$$

for all α and β. $\mathcal{S}$ is a locally convex topological vector space with the topology defined by the semi-norms that are finite according to (1.3) (cf. Schwartz [4]). Our main result is the following.