ARKIV FÖR MATEMATIK Band 3 nr 39

Communicated 13 March 1957 by Otto Frostman and L. Carleson

On the existence of a largest subharmonic minorant of a given function

By Yngve Domar

1. Introduction

Suppose that E is an open connected set in a k-dimensional Euclidean space. We say that a real-valued function $u(x)$ in E is subharmonic if it satisfies the following conditions (Radó [3], §§ 1.1, 2.3):

(i) *$u(x)$ is bounded above on every compact subset of E.*

(ii) *$u(x)$ is upper semicontinuous on E, i.e. for every real number a the set of points x, where*

$$u(x) < a,$$

is an open set.

(iii) *For every k-dimensional compact sphere $S_R(\xi) \subset E$, with centre ξ and radius R, we have*

$$u(\xi) \leq \frac{1}{S_R} \int_{S_R(\xi)} u(x)\, dx, \tag{1}$$

where S_R denotes the volume of the sphere, and where the integration is carried out with respect to the (k-dimensional) Lebesgue measure.

Let us now suppose that $F(x)$ is a given non-negative, upper semicontinuous function on E. We allow the function to assume the value $+\infty$. We shall then consider the class $\{F\}$ of all subharmonic functions $u(x)$, such that

$$u(x) \leq F(x)$$

for every $x \in E$.

It is easy to realize that if two functions $u_1(x)$ and $u_2(x)$ belong to $\{F\}$, then the same is true for the function

$$\operatorname{Max}\{u_1(x), u_2(x)\}.$$

The corresponding property holds for any finite number of functions in $\{F\}$. Our aim in this paper is to show that under certain conditions also the function

$$M(x) = \sup_{u \in \{F\}} u(x)$$

belongs to $\{F\}$, i.e. that $F(x)$ has a largest subharmonic minorant.