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A multi-dimensional prediction problem

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1. Introduction

The problem of linear prediction for a weakly stationary stochastic process has been discussed in considerable detail by Kolmogorov [3], Wiener [6] and others. Recently there has been increasing interest in the linear prediction problem for a vector-valued weakly stationary process. Aspects of this problem have been treated in a heuristic manner by Whittle [5] and analytically by Wiener [7]. The discussion in this paper is more probabilistic in orientation and some attention is devoted to the problem of computing the prediction error covariance matrix in a one-step prediction when the process is a two-vector.

2. Preliminary discussion

Let

$$x_t = \begin{pmatrix} {}_1x_t \\ \vdots \\ {}_mx_t \end{pmatrix}, \qquad t = \cdots, -1, 0, 1, \ldots, E\, x_t \equiv 0,$$

be an m-vector weakly stationary stochastic process. By this we mean that the sequence of covariance matrices $(m \times m)$

$$r_{t,\tau} = E\, x_t\, x_\tau' = r_{t-\tau}\ (^1)$$

depends only on the difference $t-\tau$. It is then well known that the sequence of covariance matrices r_t can be represented as the Fourier–Stieltjes coefficients

$$r_t = \int_{-\pi}^{\pi} e^{it\lambda}\, d\, F(\lambda) \tag{1}$$

of a matrix-valued $(m \times m)$ non-decreasing function $F(\lambda)$. The function $F(\lambda)$ is said to be non-decreasing since for any given m-vector v

$$v' \Delta F(\lambda) v = v' [F(\lambda_2) - F(\lambda_1)]\, v \geq 0,$$

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[1] Given the matrix A, A' denotes the conjugated transpose of the matrix A.