ARKIV FÖR MATEMATIK Band 3 nr 33

Communicated 6 June 1956 by T. Nagell and Otto Frostman

On a Diophantine equation of the second degree

By Bengt Stolt

§ 1. Introduction

It is easy to solve the Diophantine equation

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

with integral coefficients, in integers x and y when the equation represents an ellipse or a parabola in the (x,y)-plane. If the equation represents a hyperbola, the problem is much more difficult. For solving an equation of this type one may use either the theory of quadratic forms or the theory of quadratic fields.

T. Nagell has shown ([1]–[5]) how it is possible to determine all the solutions of the Diophantine equation

$$x^2 - Dy^2 = \pm N, \tag{1}$$

where D and N are integers and D is not a perfect square, by quite elementary methods. The author ([6]–[8]) used these methods to the equation

$$x^2 - Dy^2 = \pm 4N. \tag{2}$$

Consider the Diophantine equation

$$Au^2 + Buv + Cv^2 = \pm N, \tag{3}$$

where A, B, C and N are integers and $B^2 - 4AC = D$ is a positive integer which is not a perfect square. It is obvious that (3) can be transformed into (1) by means of linear transformations with integral coefficients. The problem of determining all the solutions of (3) in integers u and v then reduces to the problem of finding all the integral solutions x and y of (1) which satisfy certain linear congruences; see Nagell [4], pp. 214–215. However, in this way we get no general view of the solutions of (3), and it will be rather laborious to discuss the different cases which may occur.

The purpose of this paper is to show how it is possible to avoid the linear transformations and congruences. In fact, for equation (3), we shall deduce inequalities analogous to those determined by Nagell for equation (1). We shall use the notions proposed by Nagell or notions analogous to them.