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On linear estimates with nearly minimum variance

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1. Introduction

Let z be a random variable with a continuous cumulative distribution function $F[(z-\mu)/\sigma]$ which depends upon two unknown parameters μ and σ. Consider an ordered random sample

$$z_{(1)} \leqq z_{(2)} \leqq \cdots \leqq z_{(n)}$$

of z-values. If the means and covariances of the reduced order statistics

$$x_i = \frac{z_{(i)} - \mu}{\sigma} \qquad (i = 1, 2, \ldots, n)$$

are known, it is possible to find linear unbiased minimum variance estimates

$$\sum_{i=1}^{n} g_{1i} z_{(i)} \quad \text{and} \quad \sum_{i=1}^{n} g_{2i} z_{(i)}$$

of μ and σ respectively (Lloyd, 1952). These estimates may be called best unbiased estimates. A serious drawback of the solution is that in most cases it involves very time-consuming numerical calculations.

The object of this paper is to show that, under general conditions, it is possible to find a convenient approximation to the best solution which may be termed a nearly best unbiased estimate. The variance of this estimate is, as some examples will show, often very little in excess of the minimum variance. The method presupposes that the means (but not the covariances) of the variables x_i are known.

By a slight modification of the method it may be used also when neither the means nor the covariances are known. The resulting estimates will be called nearly best, nearly unbiased estimates.

Both types of estimates mentioned above may be derived from a theorem given in the next section.

2. A theorem on linear estimates

Denote by $E x_i$ and $cov(x_i, x_j)$ the means and covariances of the variables x_i $(i = 1, 2, \ldots, n)$. Put $p_i = i/(n+1)$ and $q_i = 1 - p_i$. Further, define $\lambda_i = G(p_i)$, where $G(u)$ is the inverse function of $F(x)$.