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# The remainder in Tauberian theorems II

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With 5 figures in the text

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## 1. Introduction

In an earlier paper [6] the author examined a class of Tauberian relations with exponentially vanishing remainders, i.e. relations of the form

$$\int_{-\infty}^{\infty} \Phi(x-u)\, dF(u) = O(e^{-\gamma x}) \quad \text{as } x \to \infty, \tag{0.1}$$

where $\Phi(x)$ is bounded and $F(x)$ is of bounded variation. Thus, when certain conditions are imposed on the Fourier-Stieltjes transform $f(\xi)$ of $F(x)$, it proves the validity of