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On the Diophantine equation $u^2 - D v^2 = \pm 4 N$

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Part III

§ 1. Introduction

Consider the Diophantine equation

$$u^2 - D v^2 = \pm 4 N, \tag{1}$$

where D and N are integers and D is not a perfect square. In Part I of this investigation[1] it was shown that it is possible to determine all the solutions of (1) by elementary methods[2].

Suppose that (1) is solvable, and let u and v be two integers satisfying (1). Then $\frac{1}{2}(u + v\sqrt{D})$ is called a *solution* of (1). If $\frac{1}{2}(x + y\sqrt{D})$ is a solution of the Diophantine equation

$$x^2 - D y^2 = 4, \tag{2}$$

the number

$$\frac{u + v\sqrt{D}}{2} \cdot \frac{x + y\sqrt{D}}{2} = \frac{u_1 + v_1\sqrt{D}}{2}$$

is also a solution of (1). This solution is said to be *associated* with the solution $\frac{1}{2}(u + v\sqrt{D})$. The set of all solutions associated with each other forms a *class of solutions* of (1).

A necessary and sufficient condition for the two solutions $\frac{1}{2}(u + v\sqrt{D})$, $\frac{1}{2}(u' + v'\sqrt{D})$ to belong to the same class is that the number

$$\frac{v u' - u v'}{2N}$$

be an integer.

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[1] See [1].

[2] These methods were developed by T. NAGELL, who used them for determining all the solutions of the Diophantine equation

$$u^2 - D v^2 = \pm N.$$

Nagell also proposed the notions used in this section. See [2], [3], [4], [5].