Acta Math., 195 (2005), 253–264

Decay of correlations for Hénon maps

by

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1. Introduction

Exponential mixing is an important statistical property in dynamics. It is often difficult to prove this non-linear property for a non-uniformly hyperbolic system. See Benedicks–Young [4], [5] and the references therein for the case of real Hénon maps. Here we will study a large class of polynomial automorphisms in $\mathbf{C}^k$. We note that exponential decay of correlations has been proved for polynomial-like maps and meromorphic maps in the case of large topological degree, which is the opposite of the invertible case (see [14], [8] and [9]).

Given a polynomial automorphism f of $\mathbf{C}^k$, we will extend it to a birational map of $\mathbf{P}^k$. We say that f is a *regular automorphism* in the sense of Sibony if the indeterminacy sets $I_\pm$ of $f^{\pm 1}$ (i.e. the sets of points at infinity where the birational maps $f^{\pm 1}$ are not defined) satisfy $I_+\cap I_-=\varnothing$. We recall here some properties of regular automorphisms (see [2], [1] and [13] for dimension 2 and [20] for $k\geqslant 2$). Note that when $k=2$, the regular automorphisms are finite compositions of generalized Hénon maps (see Friedland and Milnor [15]). As was shown in [15], these are the dynamically interesting polynomial automorphisms of $\mathbf{C}^2$.

The indeterminacy sets $I_\pm$ are contained in the hyperplane at infinity L_∞. When f is regular, there exists an integer s such that $\dim I_+=k-1-s$ and $\dim I_-=s-1$. We have $f(L_\infty\setminus I_+)=I_-$ and $f^{-1}(L_\infty\setminus I_-)=I_+$. Moreover, I_- is attractive for f, and I_+ is attractive for f^{-1}. Let $\mathcal{K}_+$ (resp. $\mathcal{K}_-$) denote the *filled Julia set* of f (resp. of f^{-1}), i.e. the set of points $z\in\mathbf{C}^k$ such that the orbit $(f^n(z))_{n\in\mathbf{N}}$ (resp. $(f^{-n}(z))_{n\in\mathbf{N}}$) is bounded in $\mathbf{C}^k$. Then $\mathcal{K}_\pm$ are closed in $\mathbf{C}^k$ and satisfy $\overline{\mathcal{K}}_\pm\cap L_\infty=I_\pm$. The open set $\mathbf{P}^k\setminus\overline{\mathcal{K}}_+$ (resp. $\mathbf{P}^k\setminus\overline{\mathcal{K}}_-$) is the immediate basin of I_- for f (resp. I_+ for f^{-1}). If d_+ and d_- are the algebraic degrees of f and f^{-1}, respectively, then $d_+^s=d_-^{k-s}>1$. In particular, we have $d_+=d_-$ when $k=2s$.

By $T_\pm$, we denote the *Green currents of bidegree* $(1,1)$ associated to $f^{\pm 1}$ (see