Ark. Mat., **33** (1995), 357–376

Linear analysis of quadrature domains

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1. Introduction

A recent study of the L-problem of moments in two real variables [16] has revealed that its extremal solutions coincide with the characteristic functions of all bounded quadrature domains in the complex plane. The main technical tool in obtaining this result was the theory of linear bounded Hilbert space operators with rank-one self-commutator. The aim of the present paper is to isolate and present in more detail the relationship between quadrature domains and this class of operators naturally attached to them, without any explicit reference to the original moment problem.

The identities which relate these two categories of objects are rather simple and constructive. They resemble very much some one variable formulae in the spectral theory of self-adjoint operators. Although this paper is not intended to be related to applied mathematics, we have the feeling, partially based on this comparison, that the basic formulae of this paper will be accessible in the future to a numerical approach, with benefits both for operators with rank-one self-commutator as well as for quadrature domains.

First, without entering into technical details, a few definitions and general remarks are in order. Let Ω be a bounded domain of the complex plane bounded by finitely many piece-wise smooth boundaries. The domain Ω is said to be of *quadrature* for the class $L^1_a(\Omega)$ of all integrable analytic functions in Ω, if there is a distribution u with finite support in Ω which satisfies:

$$\int_\Omega f(z)\,dA(z)=u(f),\quad f\in L^1{}_a(\Omega). \tag{1}$$

Here and throughout this paper dA stands for the planar Lebesgue measure.

([1]) Paper supported by NSF Grant DMS-9201729