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Commutators and interpolation methods

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1. Introduction

Recently R. Rochberg, G. Weiss, B. Jawerth, N. J. Kalton, M. Cwikel and M. Milman (cf. [RW], [JRW], [CJM], [CJMR] and [K]) have obtained interpolation theorems for commutators of bounded linear operators and certain operators Ω, generally unbounded and nonlinear, associated with an interpolation method for both the complex and the real case, with interesting applications to classical analysis.

In [RW] Rochberg and Weiss developed the study of these commutators for spaces obtained by complex interpolation. A similar analysis was carried out for the real method by Jawerth, Rochberg and Weiss in [JRW], where they noticed that, although there are strong analogies between the two cases, the details are very different.

The purpose of this paper is to set up a unified method of both theories. Our analysis leads to a simple approach to commutator theorems, giving the precise rôle that cancellation plays in the theory.

We set a general frame by considering pairs of interpolation methods with some nice "compatibility conditions" having in mind the two basic examples of [RW] and [JRW]:

In the complex case, the pair of interpolation methods is associated to the functionals δ_θ and δ'_θ (cf. [S] or [CC]) and the Ω-operator is defined by $\Omega a = h'_a(\theta)$, where h_a is "almost optimal" among all f such that $f(\theta)=a$.

Similarly, in the real J-method, the corresponding couple of functionals is

$$\int_0^\infty u(t)\,\frac{dt}{t} \quad \text{and} \quad \int_0^\infty (\log t)u(t)\,\frac{dt}{t},$$

and $\Omega a = \int_0^\infty (\log t) h_a(t)\,(dt/t)$, with $\int_0^\infty h_a(t)\,(dt/t) = a$.

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