

Extremal rational elliptic surfaces in characteristic p . II: Surfaces with three or fewer singular fibres

William E. Lang

Introduction

In this paper, we complete the classification of extremal rational elliptic surfaces in characteristic p , begun in [3].

In [4], Miranda and Persson classified all rational elliptic surfaces over the complex numbers such that the Mordell–Weil group of the generic fibre is finite. They called these surfaces *extremal* rational elliptic surfaces. They found 16 families of such surfaces. All but one of these families have only one member, and the exceptional family depends on one parameter.

In our first paper on extremal rational elliptic surfaces in characteristic p , we classified those where the singular fibres are semi-stable. These are the characteristic p analogues of the surfaces studied by Beauville in [1], and we called them Beauville surfaces.

In this paper, we classify all other extremal rational elliptic surfaces. The classification is identical to the classification in characteristic zero in all characteristics except two and three. (There is one exceptional case in characteristic five.) The classification in characteristics two and three looks quite different. This is due to the presence of a wild ramification term in the formula of Neron–Ogg–Shararevich, which appears only in these characteristics.

Here is a plan of the paper. In Section 1, we give the preliminary results on extremal rational elliptic surfaces that we need. (Almost all of these results appeared in [3].) In Section 2A, we classify all possible singular fibres on rational elliptic surfaces with section in characteristic two, taking into account the extra term in the Neron–Ogg–Shararevich formula. It is hoped that this list may be useful for other purposes. In Section 2B, we use the results of Section 2A (together