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Regularity conditions on parabolic measures

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1. Introduction

We consider second-order parabolic differential operators of the following form:

$$L = L_t - \frac{\partial}{\partial t} = \frac{1}{2} \sum_{i,j=1}^{n} a_{ij}(t,x) \frac{\partial^2}{\partial x_i \partial x_j} - \frac{\partial}{\partial t}.$$

Suppose that L is uniformly parabolic on $[s,\infty)\times\mathbf{R}^n$ for every $s>0$, i.e., we suppose that there exist positive constants $C(s)$ such that for each $(t,x)\in[s,\infty)\times\mathbf{R}^n$

$$C(s)|\xi|^2 \le \sum_{i,j=1}^{n} a_{ij}(t,x)\xi_i\xi_j \le \frac{1}{C(s)}|\xi|^2 \quad \text{for every } \xi\in\mathbf{R}^n.$$

We assume that the matrix $(a_{ij}(t,x))_{i,j}$ is symmetric for every t, x.

Suppose that the coefficients of L are locally Hölder continuous on $(0,\infty)\times\mathbf{R}^n$. Then the initial value problem

$$Lu = 0 \quad \text{for } t > s, \quad u(s,\cdot) = h$$

is uniquely solvable for every $s\ge 0$ and every bounded continuous function h.

The parabolic operator $L_t - \partial/\partial t$ generates a diffusion process with decreasing time parameter. That process can be characterized by the property that

$$t \mapsto v(t, X_t)$$

is a supermartingale (or a submartingale, resp.) for every function v which is continuously differentiable with respect to the time variable, twice continuously

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