ALMOST PERIODIC FUNCTIONS ON SEMIGROUPS

BY

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1. Introduction

This paper is devoted to the extension, to certain commutative topological semigroups, of the fundamental approximation theorem for almost periodic functions on groups. Several definitions are necessary before we are able to state our results.

If S is a commutative topological semigroup,[2] we shall denote by $C(S)$ the Banach algebra of all bounded continuous complex valued functions on S supplied with the norm $\|\cdot\|$ defined by

$$\|f\| = \sup_{\sigma \in S} |f(\sigma)|.$$

For each σ in S and f in $C(S)$, the translated function σf in $C(S)$ is defined by

$$\sigma f(\tau) = f(\tau + \sigma), \quad \tau \in S.$$

A function f in $C(S)$ is called *almost periodic* if the set $\{\sigma f : \sigma \in S\}$ of translates of f is conditionally compact[3] in $C(S)$. We shall denote by $A(S)$ the collection of all almost periodic functions on S. It is simple to check that $A(S)$ is a closed translation invariant subalgebra of $C(S)$.

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[2] We call S a *commutative topological semigroup* if it is a commutative semigroup having an identity element, supplied with a topology in which the map $(\sigma, \tau) \to \sigma + \tau$ from $S \times S$ to S is continuous. In the terminology of [6], S would be a commutative topological semigroup with jointly continuous addition. Subsemigroups need *not* have identities.

[3] $C(S)$ will always be considered to be topologized with the norm topology, that is, the topology of uniform convergence. We shall use "conditionally compact" to mean "having compact closure". Our definition of almost periodicity is weaker than that used by Maak in [11] and our results are disjoint from his.