AN INEQUALITY IN THE GEOMETRY OF NUMBERS

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A theorem due to L. Fejes-Tóth [2] states that if $K_1, \ldots, K_n$ are n non-overlapping convex domains each of which arises from a given convex domain K by an area-preserving affine transformation and H is a convex polygon having at most six sides which contains them then $A(H) \geqslant n h(K)$ where $A(H)$ denotes the area of H and $h(K)$ is the area of the smallest polygon having at most six sides which can be circumscribed about K.

Restricting the domains $K_1, \ldots, K_n$ to be congruent and similarly situated, C. A. Rogers [4] obtains a similar result in which H is any convex domain covering $K_1, \ldots, K_n$ and $h(K)$ is replaced by $d(K)$, the determinant of the closest lattice packing of K. Rogers's results depend on the following theorem:

THEOREM (Rogers). *Let G be a plane, strictly convex, Jordan curve containing the origin, O, of a cartesian coordinate system in its interior. Denote by $G(P)$ the translate of G which results from the translation which takes O into P. Let $P_0, P_1, \ldots, P_n = P_0, P_{n+1}, \ldots, P_{n+m}$ be points which satisfy*

(1) *the polygon, $P_0P_1 \ldots P_n$ is a Jordan polygon, Π, bounding a domain, i.e. a closed, bounded, simply-connected set, Π^*;*

(2) *the domains bounded by $G(P_{r-1})$ and $G(P_r)$ have a common boundary point if $1 \leqslant r \leqslant n$;*

(3) *the points, $P_{n+1}, \ldots, P_{n+m}$ lie in Π^*;*

(4) *the domains bounded by $G(P_r)$ and $G(P_s)$ have no interior points in common if $1 \leqslant r < s \leqslant n+m$. Then*

$$\frac{A(\Pi^*)}{4\Delta(G)} \geqslant m + \frac{1}{2}n - 1$$

where $A(\Pi^*)$ is the area of Π^* and $\Delta(G)$ is the critical determinant of G.

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