

Some results connected with Brennan's conjecture

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1. Introduction and results

1.1. An estimate of harmonic measure

In this paper we shall study some problems concerning large values of harmonic measure on the boundary of a simply connected domain in the complex plane. It is well-known that, under a proper normalization, harmonic measure of any disc of radius ρ does not exceed $\sqrt{\rho}$, and so the value of harmonic measure is considered large if it is close to this bound. The following theorem provides an estimate of the number of discs with large harmonic measure.

Theorem 1. *There exist absolute constants K and A such that for every simply connected domain Ω satisfying*

$$\infty \in \Omega, \quad \text{diam } \partial\Omega = 1$$

and any numbers $\varepsilon > 0$ and $\rho > 0$, the maximal number of disjoint discs of radius ρ and harmonic measure (evaluated at ∞) greater than $\rho^{1/2+\varepsilon}$ is at most $A\rho^{-K\varepsilon}$.

This result has some consequences for univalent functions and conformal mappings. We will indicate a couple of applications.

1.2. Integral means

Let f be a univalent function in the unit disc \mathbf{D} and t be a real number.

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