FUNCTIONS OF A COMPLEX VARIABLE WITH ASSIGNED DERIVATIVES AT AN INFINITE NUMBER OF POINTS, AND AN ANALOGUE OF MITTAG-LEFFLER'S THEOREM.[1]

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1. Introduction.

The Problem of constructing a function of a real variable which is indefinitely differentiable and has all its derivatives assigned at one or more points has been studied by BOREL and BERNSTEIN.[2] In the complex plane we may no longer require the function to be differentiable in a deleted neighborhood of the point at which the derivatives are assigned which completely surround it, unless these derivatives are subject to the restrictions as to size which hold for the derivatives of an analytic function. We may, however, require it to be analytic in a sector having the given point as its vertex. The construction of the function in this case was discussed by RITT.[3] Later BESIKOWITSCH,[4] apparently ignorant of the work of Ritt, solved the problem by a slightly different method, and also obtained some approximation theorems, proving and generalizing a theorem stated by BIRKHOFF[5] in another connection. In the present paper

[1] Presented to the American Mathematical Society, May 2, 1925.

[2] E. BOREL, Sur quelques points de la théorie des fonctions, Ann. de l'Ec. Norm., 1895, p. 38, or Fonctions de variables réelles (1905), p. 70. The problem here stated is not directly mentioned by Borel, but its solution is implicitly contained in his discussion of a related question. S. BERNSTEIN, Appendix to R. D'Adhémar, Principes de l'Analyse, vol. II, p. 272 (1913).

[3] J. F. RITT, On the Derivatives of a Function at a Point, Annals of Mathematics, 2nd series, vol. 18 (1916), p. 18.

[4] A. BESIKOWITSCH, Über analytische Funktionen mit vorgeschriebenen Werten ihrer Ableitungen. Mathematische Zeitschrift, vol. 21 (1924), p. 111.

[5] G. D. BIRKHOFF, The Generalized Riemann Problem, Proceedings of the American Academy of Arts and Sciences, vol. 49, (1913), p. 522.