AN EXTENSION OF POINCARÉ'S LAST GEOMETRIC THEOREM.

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1. Introduction.

The Crowned Memoir by POINCARÉ, »Le problème de trois corps et les équations de la dynamique», in volume 13 of the *Acta mathematica* contained the first great attack upon the non-integrable problems of dynamics. Under the direction of Professor MITTAG-LEFFLER, the *Acta mathematica* has had many remarkable articles, but perhaps none of larger scientific importance than this one. Its many ideas, in which the periodic motions took a central part, led naturally to POINCARÉ's later dynamical researches.

In a highly interesting paper, »Sur un théorème de géométrie», published shortly before his death in volume 33 of the *Rendiconti del Circolo Matematico di Palermo*, POINCARÉ showed that a certain geometric theorem (proved by him in particular cases) would carry with it the answer to some outstanding questions concerning the periodic motions. The peculiarity of the method by which I obtained a general demonstration of its truth soon afterwards,[1] and the dynamical origin of the theorem itself, have suggested the extension given here.

In thus responding to the kind invitation of Professor NÖRLUND, I desire to render homage to Professor MITTAG-LEFFLER, especially because of the inspiring tradition which he has established for the *Acta mathematica*.

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[1] Proof of Poincaré's Geometric Theorem, *Transactions of the American Mathematical Society*, volume 14; or see a translation in volume 42 of the *Bulletin de la Société Mathématique de France*.