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Non straightenable complex lines in $\mathbf{C}^2$

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Abhyankar and Moh, in [1, (1.6), p. 151], and M. Suzuki, in [11, §5], proved that if P is a polynomial embedding of $\mathbf{C}$ into $\mathbf{C}^2$, then there exists ψ, a polynomial automorphism of $\mathbf{C}^2$, such that $(\psi\circ P)(\mathbf{C})=\mathbf{C}\times\{0\}$. The corresponding and much easier result has been proved for polynomial embeddings of $\mathbf{C}$ into $\mathbf{C}^n$, for $n\geq 4$ by Z. Jelonek [7] (more generally, Jelonek treats the case of embeddings of $\mathbf{C}^k$ into $\mathbf{C}^n$, for $n\geq 2k+2$). The case of polynomial embeddings of $\mathbf{C}$ into $\mathbf{C}^3$ seems open. See also [8].

The main goal of this paper is to show that the above results do not generalize to holomorphic embeddings of $\mathbf{C}$. Another goal is an interpolation theorem (Proposition 2 below).

Proposition 1. *Let $n>1$. There exists a proper holomorphic embedding $H:\mathbf{C}\to\mathbf{C}^n$ such that for no automorphism ψ of $\mathbf{C}^n$, $(\psi\circ H)(\mathbf{C})=\mathbf{C}\times\{0\}\subset\mathbf{C}^n$.*

Notice however that it has been proved in [4, (4.1)] that for every $R>0$ and $\varepsilon>0$ there exists ψ, an automorphism of $\mathbf{C}^n$, such that $|(\psi\circ H)(\zeta)-(\zeta,0)|\leq\varepsilon$ for every $\zeta\in\mathbf{C}$, $|\zeta|\leq R$. So, compact subsets of the complex line $H(\mathbf{C})$ can be "approximately straightened".

The above proposition has been known for some time, for $n\geq 3$. In [4, (7.8)] this is pointed out as being (non stated but) clear in [10]. For $n=2$, we keep the same approach as in [10]. But we can now take advantage of the ground breaking work by Andersén and Lempert [2], as further developed in [4].

1. Proof of Proposition 1

Proposition 1 is an immediate consequence of the following two propositions:

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