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Boundedness, compactness, and Schatten p-classes of Hankel operators between weighted Dirichlet spaces

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1. Introduction and main results

In this paper, we study the small and big Hankel operators from one weighted Dirichlet space to another. We characterize the analytic symbols for which these operators are bounded, compact or belong to the Schatten p-classes for a certain range of p. The endpoints of this range are also discussed.

Let $\mathbf{D}$ be the unit disk in the complex plane. Let $dA(z)=1/\pi\, dx\, dy$ be the normalized area measure on $\mathbf{D}$. For $\alpha<1$, set

$$dA_\alpha(z)=(2-2\alpha)(1-|z|^2)^{1-2\alpha}\, dA(z).$$

The Sobolev space $L^{2,\alpha}$ is the Hilbert space of functions $u:\mathbf{D}\to\mathbf{C}$, for which the norm

$$\|u\|_\alpha=\left(\left|\int_{\mathbf{D}} u(z)\, dA_\alpha(z)\right|^2+\int_{\mathbf{D}}(|\partial u/\partial z|^2+|\partial u/\partial \bar z|^2)\, dA_\alpha(z)\right)^{1/2}$$

is finite. The weighted Dirichlet space D_α is the subspace of all analytic functions in $L^{2,\alpha}$. (This scale of spaces includes the Bergman space ($\alpha=-\frac{1}{2}$), the Hardy space ($\alpha=0$) and the classical Dirichlet space ($\alpha=\frac{1}{2}$).) The orthogonal projection, P_α, from $L^{2,\alpha}$ onto D_α can be understood as the integral operator represented by

$$P_\alpha(u)(w)=\int_{\mathbf{D}} u(z)\, dA_\alpha(z)+\int_{\mathbf{D}}\frac{\partial u}{\partial z}(z)\overline{\frac{\partial}{\partial z}K_\alpha(z,w)}\, dA_\alpha(z).$$

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