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Removability theorems for solutions of degenerate elliptic partial differential equations

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1. Introduction

Removable singularities for Hölder continuous harmonic functions are completely known, see [C₁], [C₂, p. 91] and [KW].

Theorem A. *Let Ω be an open set in $\mathbf{R}^n$ and let E be a relatively closed subset of Ω. Then E is removable for harmonic functions of $\Omega\setminus E$ which are locally Hölder continuous in Ω with exponent $0<\alpha<1$ if and only if the $(n-2+\alpha)$-dimensional Hausdorff measure of E is zero.*

We recall that a function $u:\Omega\to\mathbf{R}$ is said to be locally Hölder continuous in Ω with exponent $0<\alpha\le 1$ if for each compact subset K of Ω there is $M<\infty$ such that

$$|u(x)-u(y)|\le M|x-y|^\alpha \tag{1.1}$$

for all x and y in K.

In this paper we consider an analogous question for solutions of second order degenerate elliptic partial differential equations. For linear equations we refer the reader to [HP]. We call a function u $\mathcal{A}$-harmonic if u is a continuous weak solution of the equation

$$\operatorname{div}\mathcal{A}(x,\nabla u(x))=0 \tag{1.2}$$

with $|\mathcal{A}(x,\xi)|\approx|\xi|^{p-1}$, $p>1$. For the exact requirements on the mapping $\mathcal{A}$ we refer the reader to Section 3. Here we point out that the prototype of equation (1.2) is the p-harmonic equation

$$\operatorname{div}(|\nabla u|^{p-2}\nabla u)=0. \tag{1.3}$$