Ark. Mat., **31** (1993), 197–208

Special results in adjunction theory in dimension four and five

Mauro C. Beltrametti and Andrew J. Sommese

Introduction

Let $L^\wedge$ be a very ample line bundle on an n-dimensional projective manifold $(X^\wedge, L^\wedge)$. Assume $n = \dim X^\wedge \geq 3$. In [1], Beltrametti, Fania, and Sommese give a very explicit structure theory for pairs $(X^\wedge, L^\wedge)$ as above, and such that the Kodaira dimension $\varkappa(K_{X^\wedge} + (n-3)L^\wedge) < n$ and $n \geq 6$. Moreover, if $n \geq 6$ and $\varkappa(K_{X^\wedge} + (n-3)L^\wedge) = n$ it is shown there is a very simple birational morphism $f: X^\wedge \to X$ with X having at most 2-factorial isolated terminal singularities, and $K_X + (n-3)L$ nef and big where $L = (f_* L^\wedge)^{**}$ is at worst 2-Cartier.

Partial results are given for dimensions $n = 3, 5$ by Beltrametti, Fania, and Sommese [1] and for $n = 4$ by Fania and Sommese [4].

In this paper we extend the structure theory of Beltrametti, Fania, and Sommese [1] to $n = 5$ in the same form as the structure theorem when $n \geq 6$. See (1.1) and (1.2) for a statement. We also extend the structure theorem if $n = 4$, to cover pairs $(X^\wedge, L^\wedge)$ where $\varkappa(3K_{X^\wedge} + 4L^\wedge) < 4$. If $\varkappa(3K_{X^\wedge} + 4L^\wedge) = 4$ there is a very simple morphism $\psi: X^\wedge \to Z$ with Z having at most Gorenstein, 2-factorial, isolated terminal singularities, and $3K_Z + 4L$ nef and big where $L = (\psi_* L^\wedge)^{**}$ is at worst 2-Cartier. See Theorems (2) and (2.5) for a complete statement.

We would like to thank the Max-Planck-Institut für Mathematik in Bonn for its support. This paper was conceived and mainly worked out during the authors stay at the Max-Planck-Institut in July, 1991. The second author would like to thank the National Science Foundation (NSF Grant DMS 89-21702) for its support.

After this paper was finished we received a preprint of T. Fujita, *On the Kodaira energy and adjoint reduction of polarized manifolds*, (which will appear in Manuscripta Math.) that overlaps with our paper. In particular T. Fujita has shown that in case (1.1.2) of Theorem (1.1), $(X, \mathcal{K})$ is the projective cone over the Veronese 4-fold, $(\mathbf{P}^4, \mathcal{O}_{\mathbf{P}^4}(2))$.