Acta Math., 176 (1996), 171–243

Multiplicities of recurrence sequences

by

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1. An introduction

We will study equations

$$\sum_{i=1}^{r} f_i(m)\alpha_i^m = 0 \qquad (1.1)$$

in the variable $m \in \mathbf{Z}$. Here the f_i are nonzero polynomials with complex coefficients of respective degrees k_i $(1 \leqslant i \leqslant r)$ and we put

$$k_1 + \ldots + k_r + r = q. \qquad (1.2)$$

We suppose that the α_i are nonzero elements of a number field K with

$$[K:\mathbf{Q}] = d \qquad (1.3)$$

and that moreover for each pair i, j with $1 \leqslant i < j \leqslant r$,

$$\alpha_i/\alpha_j \text{ is not a root of unity.} \qquad (1.4)$$

We prove

THEOREM 1.1. *Assume that we have* (1.2), (1.3), (1.4). *Then equation* (1.1) *has not more than*

$$d^{6q^2} 2^{2^{28q!}} \qquad (1.5)$$

solutions $m \in \mathbf{Z}$.

Results on equations (1.1) have been derived recently in [14] and shortly afterwards in [12]. However in both papers the bound for the number of solutions is only "semi-uniform", as it depends upon q, d and moreover upon ω, which is defined as the number

Written with partial support of the Italian Consiglio Nazionale delle Ricerche.