THE DEFINITE INTEGRAL $\int_{-\infty}^{\infty} \frac{e^{ax^2+bx}}{e^{cx}+d} dx$ AND THE ANALYTIC THEORY OF NUMBERS.

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§ 1.

Introduction.

Professor Siegel[1] in a memoir recently published dealing with the manuscripts left by Riemann has pointed out that Riemann dealt with some integrals of the type

$$I = \int_{-\infty}^{\infty} \frac{e^{at^2+bt}}{e^{ct}+d} dt$$

in his researches on the zetafunction. Not only can the usual functional equation be thus found, but also an asymptotic formula is obtained for the zetafunction of which the first term gives the well known approximate functional equation due to Hardy and Littlewood.[2]

Kronecker's evaluation[3] of the Gauss's sums by special integrals of this kind is classic. Not so well known is his evaluation[3] of the integral

$$\int_{-\infty}^{\infty} \frac{e^{\pi i t^2/n}}{\cosh \pi t} dt$$

[1] Siegel 45—48.

[2] Hardy and Littlewood (3), (4), (5).

[3] Kronecker (1), (2).