

# ANALYTIC THEORY OF LINEAR $q$ -DIFFERENCE EQUATIONS.

By

W. J. TRJITZINSKY  
of EVANSTON, ILL., U. S. A.<sup>1</sup>

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§ 1. **Introduction.** The subject of this paper is to develop the analytic theory of a  $q$ -difference system

$$(1) \quad \begin{aligned} Y(qx) &= A(x) Y(x), & Y(x) &= (y_{ij}(x)), \\ A(x) &= (a_{ij}(x)), & |A(x)| &\neq 0 \quad (i, j = 1, \dots, n) \end{aligned}$$

or, which is an essentially equivalent matter, the analytic theory of a single  $q$ -difference equation

$$(1a) \quad \begin{aligned} L_n(y) &\equiv y(q^n x) + a_1(x)y(q^{n-1}x) + \dots + a_n(x)y(x) = 0 \\ &(a_n(x) \neq 0). \end{aligned}$$

It is assumed that the coefficients  $a(x)$  in (1) or (1a) are analytic for  $|x| \leq \varrho$ , being representable for these values of  $x$  as follows

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<sup>1</sup> The author began this work while he was a National Research Fellow at Harvard University.